

Task description

Pupils Investigate patterns in the number of nails on a geoboard used to form a square.

Suitability	National Curriculum levels 6 to 7
Time	30 minutes to 1 hour
Resources	Ruler, pencil, calculator and 1 cm squared dotted paper

Key Processes involved

- **Representing:** Draw squares on centimetre squared dotty paper.
- **Analysing:** Count the 'nails' round the edge and in the centre of their squares. Collect results into a table and look for patterns in the numbers.
- **Interpreting and evaluating:** Use their patterns to answer the questions on the sheet.
- **Communicating:** Explain what patterns they find and how they have used them to answer the questions.

Teacher guidance

Check that pupils fully understand the context, for example, show them a geoboard with a square on it and ask:

- *Can you see that the third square has 12 nails round the edge and 4 in the middle?*
- *Read all the questions and answer them as carefully as you can.*
- *Show all your working so that I can understand your thinking.*

Pupils may tackle the task in different ways, but they might be expected to:

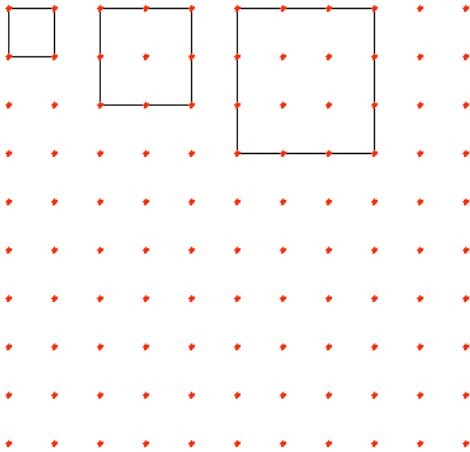
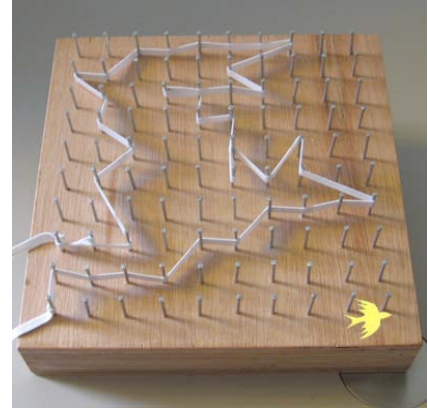
- *Draw squares*
- *Count the nails on the edges and inside the squares*
- *Put their results into a table*
- *Find patterns in their results*
- *Use these patterns when answering the questions on the sheet*

Geoboard Squares

This is a picture of a Geoboard. You can make shapes on the board by wrapping a rubber band around the nails.

The figure below is a pattern of squares.

- The first square has 4 nails; the length of its sides is 1
- The second square has 8 nails on the edges and 1 nail inside; the length of its side is 2
- The third square has 12 edge nails and 4 nails inside

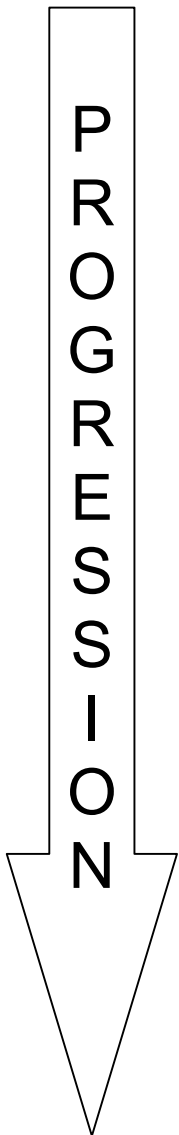


You could continue this pattern and count the number of nails on the edges and inside each square.

1. How many edge nails and how many nails inside are there for a square with 12 nails on one side?
2. Explain whether or not there is a square which has 2025 nails inside it.
3. Is there a square which has the same number of edge nails as inside nails?

Assessment guidance

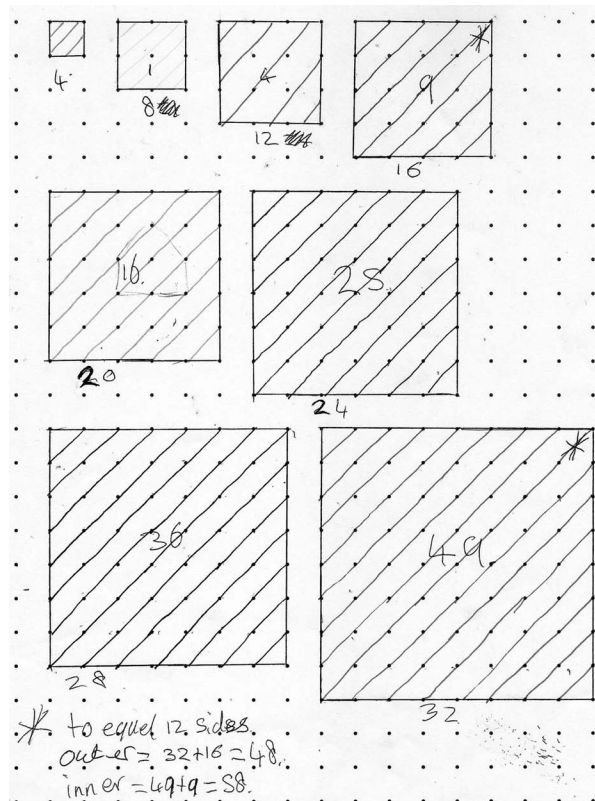
Progression in Key Processes



Representing	Analysing	Interpreting & Evaluating	Communicating
Results generated, i.e. drawings	Counting and calculating	Looking for patterns in results	Clarity and completeness of written work
Draws some squares but makes no attempt to find patterns in the numbers; does not draw the 12 th square. Pupil A	Counts the nails inside and round the sides of their squares. Pupil A	Draws several squares and counts the nails but makes no attempt to find patterns from their numbers. Pupil A	Communicates the work adequately, but with gaps and/or omissions. Pupil A
Draws some squares, including the 12 th square; makes no attempt to find patterns in the numbers. Pupil B	Finds the number of edge and inside nails for the 12 th square, but no patterns in the numbers. Pupil B	Realises that drawing the 12 th square is a good idea and is able to do this correctly, but makes no attempt to find patterns. Pupil B	Communicates the work clearly and the reasoning can be followed. Pupil B
Draws squares and finds some patterns, but is not able to use what they find to answer the questions. Pupil C	Finds the number of edge and inside nails for the 12 th square and rules to find the number of edge and inside nails. Pupil C	Correctly finds the values for the 12 th square and some patterns in their numbers. Pupil C	Communicates the work clearly and the reasoning can be followed easily. Pupil C
Draws squares and finds n th terms for the sequences of numbers. Uses these rules to answering the questions. Pupil D	Finds the rules and uses them to answer the questions. Pupil D	Successfully finds the n th terms and uses them. Pupil D	Explains the work clearly and considers using other shapes. Pupil D

Sample responses

Pupil A



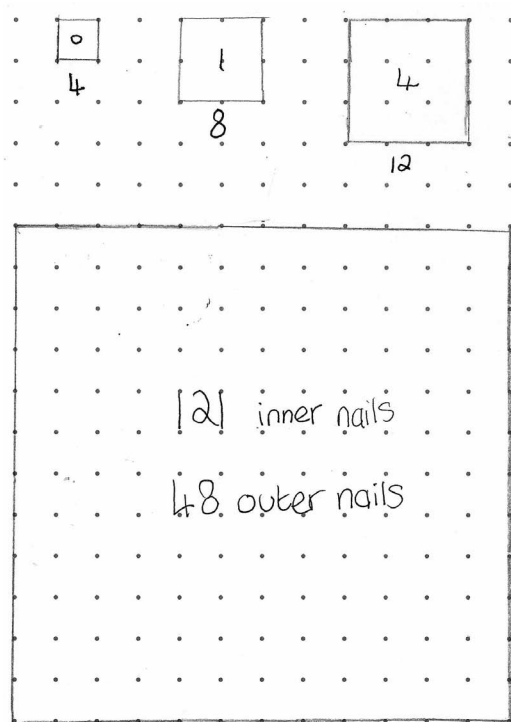
Comments

Pupil A has drawn several squares and counted the nails inside and round the edge. He did not draw the twelfth square and made an incorrect attempt to work out the nails inside and on the edge of it.

Probing questions and feedback

- How did you get the numbers for the 12th square? Are you sure they are correct?
- Can you see any pattern in the numbers you have found?

Pupil B



Comments

Pupil B has drawn several squares, including the twelfth one, and correctly counted the inside and edge nails. She provided no explanation and attempted no further work

Probing questions and feedback

- *How many edge and inner nails do you think the fourth square will have?*
- *Can you see any patterns in your numbers?*
- *Would it help to put the numbers in a table?*

Pupil C

A. I drew a square with sides 12 on paper.
→ Inner nails → 121
Edge nails → 48

B. → $45 + 1 = 46$
I got this by finding the square route
of 2025

C. → NO! because there is always more
nails on the outside than in the
middle.
Also you always times by 4 to get the
outside amount of Nails and square
for the inside!

Comments

Pupil C has found the number of inside and edge squares for the twelfth square and the rule for the inside squares which she has then used. Her final conclusion is inadequate and she has not shown and explained her findings fully.

Probing questions and feedback

- Can you explain why you found the square root of 2025 and added one to the result to get the answer to part B?
- Are you sure that there are always more nails on the outside than the inside? Do you think that making a table of results would help to show what happens?
- Can you work out the n^{th} terms for the numbers of nails?

Pupil D

side nails	inner nails	side length
4	0	1
8	1	2
12	4	3
16	9	4
20	16	5
24	25	6
28	36	7
32	49	8
36	64	9
40	81	10
44	100	11
48	121	12

- the side nails go up in fours or you times the side length by 4 giving $4n$
- the inner nails are square numbers or the side length before the number times its self. For example if you wanted to work out the inner nails if the side length is 7, you do the number before (6) and times it by itself ($6 \times 6 = 36$). For n it is $(n-1)^2$.

a. side nails - 48 inner nails - 121

b. 2025 is a square number $\sqrt{2025} = 45$
 $45 + 1 = 46$ the square will have a side length of 46.

c. NO! side length 5 side nails = 20 ← bigger like 1, 2, 3 and 4
 inner nails = 16
 side length 6 side nails = 24 ← smaller like 7, 8, 9, 10, ...
 inner nails = 25

To be the same it is between 5 and 6 !

Comments

Pupil D has drawn up a table of his results and used it to find n^{th} terms and answer the questions.

Probing questions and feedback

- Using your n^{th} terms, can you make an equation for the number of edge nails to equal the number of inside nails, then try to solve it (eg by trial and improvement)?
- Could you try finding rules for shapes other than squares?